# Marx's Theory of Crisis in the Context of Financialization. Analytical Insights on the Current Crisis. II

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## Abstract:

*Lain by the fact that the present crisis was not preceded by a sharp decline in the rate of profit, Marxist and heterodox economists, investigate its' causes in structural and institutional factors emerging in a «new phase» of capitalism dominated by «financialization» of capital. This paper argues that, in Marx, a sharp decline in the rate of profit is not a prerequisite for a crisis to emerge, if the rate of profit is already low. We argue further, that «financialization» of capital resulted, following the «great stagflation» of the 70s, from a strategy to battle low profitability by suppressing interest rates in order to increase the «rate of profit of enterprise». We show that this policy is, in the end, limited by the «rate of profit» and when the limit is reached the system collapses as it did in 2007. These analytical conclusions suggest that bank recapitalization will have restricted impact on output and employment because debts are already too high and profits too low for these funds to end up supporting corporate investment. Alternative policies should be applied otherwise a long period of capital impairment and high unemployment lies ahead of us.*

**Keywords**: Crisis, Rate of Profit, Rate of Profit of Enterprise, Financialization
**JEL classification:** E44, B22, B51

## I. Introduction – The Issue

Six years have passed since the failure of Bear Sterns which marked the beginning of the current crisis. Although trillions of government and central bank money were disbursed to securitize financial capital over this period and contrary to mainstream wisdom (Friedman & Swardz 1971[1]), mounting unemployment, bank, corporate and sovereign failures persistently prevail, revealing beyond doubt that we are faced with a depression. The idea that the current crisis resulted from an extreme event in the financial sector, the appearance of a black swan (Thaleb 2009) inside a flock of white swans, is greeted with increasing skepticism among economists[2]. In order to explain the duration of the crisis, to reduce it to a set of national episodes arising from different independent causes and most importantly to justify austerity policies, mainstream economics presented a set of theories suggesting that high state deficits and sovereign debt (Rogov and Reinhard. 2010) undermine growth. But economic data indicates otherwise, recent econometric works (Blashard &Leigh,2013) have shown that the negative association of state deficits with growth was based on false assumptions over the magnitude of the fiscal multiplier in growth and depression. For the correlation between the sovereign debt/ GDP ratio and growth the findings were even more embarrassing, it was shown that the negative association was almost 20% off due to a particular handling of the data itself (Hendron, Ash & Polin 2013). But, most importantly, econometricians argue (Panizza & Presbitero 2013) that sovereign debt data indicates causality running from weak growth to high debt/GDP ratios rather than the opposite, suggesting a different cause for the prevalence of the current depression. This brief account shows, among other issues, that the debate over the cause of the current crisis is not merely academic since it justifies or condemns policies affecting the lives of billions al-lover the globe.

History has taught us that capitalist economies experience periods of prosperity followed by depressions with almost periodical recurrence (Singer - Kerel 1970), this suggesting an underlying pattern inherent in the mode of production. Marx argued one and a half century ago that profit motivated growth, which characterizes capitalism, is inherently contradictory, the contradictions expressed in the long term tendency of the rate of profit

[1]The main idea in the monetarist approach is that the Great Depression could have been avoided if the Federal Reserve Bank had acted as lender of last resort. This is why the sited authors refer to the Great Depression as "Great Recession".

[2]Even mainstream economists reject this line of reasoning (Roubini N. 2011). Roubini and Minh argue that the recurrence of crises is so intense that crises events should be referred to as «white swans» rather than black.

to fall. Subsequently Henryk Grossman (Grossman 1929) analytically proved that a declining rate of profit ultimately reduces profits (mass of profit) and argued that this is the crisis theory in Marx. Later works (Mandel 1980) showed correlation between the rate of profit and «long waves» in capitalist production. Shaikh 1992 provided theoretical and empirical evidence supporting the idea that a low or declining rate of profit is the cause of these «long waves».

However, the particular actualities which preceded the current crisis spread doubt whether this scheme is valid in explaining the present episode in the history of capitalism. Profit rates in most major economies were roughly stable over the past thirty years[3], the period between the end of the last depression and the outburst of the current one. Heterodox economists engaged in a vigorous debate over the mater, a good part of them investigating the causes of the crisis (Lapavitsas 2009)[4] in systemic changes, effected by the «financialization» of capital, the later undoubtedly the most important novel aspect of contemporary capitalism. The argument is similar in methodology and sometimes in analytical conclusions to that of Hilferding (Hilferting 1910) and Lenin (Lenin 1915). In the modern version it is the autonomy of the financial sector and the fusion of finance in all aspects of life which effects qualitative changes of socioeconomic relations in rough analogy to the dominance of «Finance Capital» and the prevalence of cartels and trusts in Hilferding one century ago. But this set of theories, both past and contemporary, bear an inherent problem, they are not depression theories. When it comes down to explaining crisis they either:1) apply to disproportional growth between departments I and II, 2) forward versions of the under-consumption argument, or 3) refer to the circumstances economic, social, political and/or geopolitical which possibly led to it (Tome 2011). In the first case only the ten year cycle is considered as inherent in capitalism, depressions are ruled out, in the second capitalism is stagnant by nature and growth comes from «external sources», the third case involves possibility crisis rather than crisis theories. The bottom-line is that in the absence of a theory incorporating: inherent causes (profit motivated growth), particular actualities (stable profit rate) and novel aspects (financialization), heterodox economics are in difficulty to arrive to conclusions on the causes of the crisis and produce policy alternatives supporting political and social activism.

This paper analytically explores the potentiality of the original argument in Marx to combine all these properties. If Marx s' argument holds in this context then analytical insights on aspects of «financialization» of capital can be suggested and theoretical conclusions are drawn on the class character and effectiveness of the policies implemented.

**II. The Argument**

Which is the crucial factor that determines the turning of a «possible crisis», as elaborated in Part II of the «Theories of Surplus Value» (K. Marx 1861-3), to an actual crisis? When we refer to depressions it is the rate of profit. However, I suggest that the level rather that the dynamics of the rate of profit is crucial in this regard. In VIII following a passage where low profit rates are implied and the dynamics of the rate of profit are noted in passing Marx states: «...a rise in interest [not a decline in the rate of profit-NS] separates prosperity and its reverse...» (K. Marx 1894, p. 235). The extract implies that when the rate of profit reaches a certain limit interest rates explode, the «rate of profit of enterprise» (rate of profit less interest rate), which determines active/corporate investment, turns stagnant (Shaikh.A 1992) (K. Marx 1894) and «prosperity» turns to crisis. In Marx, a sharp reduction in the rate of profit is by no means a prerequisite for the outburst of a crisis, but low profit rates prevail prior to the crisis becoming evident.

In order to establish this argument we need to explore how profit and interest rates are associated both in normal accumulation and in a crisis. The extract which follows gives us the outline of a possible connection:

«...*it [the rate of interest-NS] "depends partly upon the rate of gross profits, partly on the proportion in which these are separated into profits of capital and those of enterprise. This proportion again depends upon the competition between the lenders of capital and the borrowers; which competition is influenced, though by no means entirely regulated, by the rate of gross profit expected to be realized»* (K. Marx 1894, p.237)[5].

Both in «The Capital» and «Theories of Surplus Value» it is gross profitability which influences the rate of interest, rather than the opposite this is why any crisis explanation based on interest and credit is ruled out as

[3] Many economists agree that the «rate of profit» was relatively stable during the pre-crisis period. Clear empirical evidence is included in: (Shaikh 2011). It should be noted, however, that following the end of the 1980 recession, profit rates never increased to their pre- 1970 levels (Lapavitsas 2009).

[4] An extensive survey is part of the sited paper by C. Lapavitsas

[5]The quotation mark in the passage refers to Marx's citation of Ramsey

superfluous[6]. Therefore it is in the determinants of the rate of profit we should look for factors influencing competition between «borrowers and lenders» and thereby the distribution between «profit of capital and profit of enterprise». As we all know, in Marx, low profit rates are coupled (actually caused by) by a high organic composition of capital, this means also that total capital advanced (constant plus variable) increases, «the productive powers of labour must be paid for»» as Marx states. At the same time the rate of profit is the ultimate regulator of profitability (mass of profit). Therefore when profits are low capital needs are great and more leverage is required for the continuation of capital accumulation, because capital is advanced and profits are realized in subsequent production periods. To put it in contemporary market terms, under this reasoning, a low rate of profit implies high «leverage ratios» especially in full capacity utilization. As we will show (section 2), a linear positive relation between the rate of profit of enterprise and the share of corporate profits implies a positive association between the rate of interest and the debt/ capital advanced ratio for non-negative nominal interest rates.

In normal accumulation the debt/capital advanced ratio can increase, because of investments both in fixed and circulating capital and interest rates increase. However, because profitability remains sufficiently high corporations are capable of building, out of their profits, adequate equity reserves to support an efficient downsizing of production. Efficient in the sense that, when interest rates rise, corporations react by reducing capacity utilization and this releases liquidity, because the part of fixed constant capital turned idle is small relative to the size of equity. The released liquidity restores the leverage ratio, interest rates decline and growth resumes. But when the rate of profit is below a certain limit corporations cannot build adequate reserves to support efficient downsizing of production. Although economic activity declines the system remains illiquid. because all funds released from commodity circulation are used up immediately as means of payment. Deteriorating confidence «...in the continuity of the reproduction process...» makes things worse since it reduces the ability of banks to build loan able reserves «because the demand for ... commercial credit [as opposed to bank credit-NS] diminishes.» (K. Marx 1894, p.330-1) since transactions are settled in cash rather than bills of exchange. Consequently in the words of Marx, «The rate of interest reaches its peak during crises, when money is borrowed at any cost to meet payments» (K. Marx 1894, p.235), the rate of profit of enterprise turns zero or negative and growth turns to stagnation or decline.

We have incorporated these insights in a growth model where accumulation depends on profit of enterprise and fluctuations represent variations in the rate of interest the latter influenced, through financial ratios, by the prevailing rate of profit which is treated as data to keep the dynamics traceable. The model exhibits very interesting properties, under certain rate of profit values it exhibits secular or chaotic growth and for different values profitability and production turns stagnant. Furthermore, the model touches on important work from Marxist economists relating to internally generated growth as presented in the «schemes of expanded reproduction» (Dumenil 1977), elaborations on the possibility of crisis theory in relation to Marx's theory of money (Folley 1984), the association of effective demand to corporate finance and the interaction of productive capacity with capacity utilization (Shaikh 1989). The main difference is that, in our context, the rate of interest is expressly determined and varies in time.

Although this model shows that crisis prevails in a low profit rate environment, even if the rate of profit does not decline further, it misses the impact of novel phenomena which followed the great depression of the 70 s', also referred to as the «great stagflation». The later, emerged from a persistent decline in the rate of profit during the preceding post-war decades. To deal with the crisis severe deregulation of the labour market and the demolition of welfare state were implemented, but although profit rates stopped declining they never reached pre-crisis levels. To restore growth interest rates were suppressed, through low central bank intervention rates and severe financial market deregulation, in order to boost the rate of profit of enterprise. Growth resumed but was limited from restricted profitability, financial sector growth however exploded, because of the deregulated markets. This phenomenon is referred to as «financialization» of capital.

Thus, I argue that, bank deregulation was not the result of neo-liberal market fundamentalism but a strategy to battle low profitability. However, financialization modified economic behaviour, corporations extended their balance sheets with various debt instruments to inflate their otherwise low equity returns, workers tried to maintain and improve their standard of living, which was setback from labour market deregulation, through cheap credit, sovereigns used low debt service costs to boost economic activity through budget deficits and banks,

[6] I refer to the known citation of Grossman (Grossman.H 1929) from the «Theories of Surplus Value»:

«In investigating why the general possibility of crisis turns into a real crisis, in investigating the conditions of crisis, it is therefore quite superfluous to concern oneself with the forms of crisis which arise out of money as means of payment [credit— HG]). This is precisely why economists like to suggest that this obvious form is the cause of crises». (K. Marx 1861-3, 514-5)

supported by low central bank intervention rates, promoted a whole set of new assets, financial «products», markets and intermediaries in order to increase the velocity of circulation and thereby extend their balance sheets without increasing interest rates. Financial risk was fused to the whole society and the lives of billions all-over the globe became directly dependent on the functioning of the financial system and the movement of financial markets.

These novel phenomena and the ability of the financial sector to influence the velocity of circulation, drove a part of heterodox economists to develop arguments supporting the idea that financialization does not necessarily imply weak production and consequently a possible crisis, in «financialized capitalism», may not result from low or declining profitability but also from structural factors inside the financial sector (Lapavitsas 2009), the current crisis falling in the second category. However, the whole argument rests on the assumption that households, the state and especially corporations will be eager to extend their balance sheets and support financial sector growth in all circumstances. For corporations, as argued above, financial needs are greater the lower the rate of profit, for workers or employees in general debt needs are inversely proportionate to their incomes which are dependent on growth, finally for the state good part of budget deficits reflect trade balances, as suggested by the «twin deficit» hypothesis (Godley. & Lavoie 2007[7])and the state of the economy. Trade balances, in turn, being heavily dependent on the competitive position of national economies. Therefore, the growth and development of the financial sector is dependent upon capitalist production and disproportional growth of finance indicates weak production.

Taking these insights into account, the initial model was modified to picture a state where both and profit and interest rates are kept constant while the velocity of circulation is boosted to trigger growth in an otherwise stagnant economy. We show that this pattern is limited by profitability and when the limit is reached the system collapses as it did in 2007 triggering the current depression.

**III. Paper Structure - Model Formalization**

The paper is organized as follows: Section 1 provides notation and accounting definitions together with their analytical implications. Section 2 analyzes the assumptions of the model. Section 3 includes the solution of the original model, stability and fluctuations analysis. Section 4 provides simulation results of the main model variables in growth and stagnation. Section 5 modifies the original model to incorporate the special policies and contradictions that led to the present depression. The final section summarizes the findings and policy implications.

**1. Notation and definitions:**

Following Marx, one period lag in profit realization is assumed. Production takes time capital is advanced at the beginning of the production period whereas profits are realized at the end of the period. Consequently, the rate of profit is the ratio of next period profits to total capital advanced:

$$r = \frac{\mathrm{Pr}_{t+1}}{K_t} \quad \text{(1.1) where Pr is gross profit and K capital advanced}$$

The rate of profit is held constant by assumption. Irrespective of the previous discussion over profit rate dynamics, this is a legitimate abstraction since the rate of profit is a «slow» variable in Marxist economics, it changes much slower than interest and prices, thus it is reasonable to appear as data in a model which investigates profit growth against interest rate dynamics. However, there are further analytical implications, because under a variable profit margin on costs, the prevailing rate of profit will deviate from its' gravitation point, or declining trend the motion reflected in variations of capacity utilization (Shaikh. A 1992). Assuming a constant profit margin on costs implies that the basic rate of profit (the gravitation point or trend of the rate of profit) will always equal the prevailing rate of profit. This assumption is equivalent to ruling out counteracting tendencies on the profit rate, such as wage reductions relative to prices, in order to explain the turning of normal accumulation to depression. Formal proof is provided in appendix 1.

We define capacity utilization as the ratio of capital advanced to total assets or equity plus liabilities, where liabilities are reduced to debt by abstracting from commercial credit.

$$u_t = \frac{K_t}{L_{t+1} + EQ_t} \quad \text{(1.2) Where L stands for debt and EQ stands for equity}$$

[7] The twin deficit hypothesis, suggesting that fiscal deficits couple trade deficits holds only in the event that investment equals savings in the private sector. However, trade balance deficits remain an important part of fiscal deficits.

Of course this not the definition of capacity utilization but can be derived from the ratio (Q/Q*) (actual output (Q) over capacity output (Q*) (Shaikh and Moudoud 2004) which is the definition under specific assumptions.

Since a constant profit rate implies also a constant capital output ratio, the following relation holds:

$\frac{Q_{t+1}}{Q^*_{t+1}} = \frac{K_t}{K_t^*}$ Where, K* stands for full capacity capital at time t

Assuming, that the asset side at the beginning of the production period stands as follows:

$$A_t = FC_t + W_t + M_t + INV_{t-1} = FCu_t + FCi_t + W_t + M_t + INV_{t-1} = K_t + FCi_t + INV_{t-1}$$

Where: FC stands for fixed capital (at purchase cost), FCu for utilized fixed capital, FCi for idle fixed capital, W, M provisions made for labour and material costs respectively and INV inventory of finished goods coming from the previous period.

Furthermore, utilized fixed capital plus wages and material costs make for the amount of capital advanced (K). For equation (1.2) to hold the following relation must hold as well:

$$K^*_t = K_t + FCi_t + INV_{t-1} \Rightarrow u_t = 1 - \frac{FCi_t + INV_{t-1}}{K^*_t} \quad (1.2')$$

Therefore, equation 1.2, elaborated in eq. (1.2') implies that capacity utilization reacts to capital stock reflected in fixed capital remaining idle (or becoming over-utilized) and aggregate demand reflected in inventory outstanding (or advances against future production). Cost price increases are partly reflected on capacity utilization, since inventory is valued at past production cost and fixed assets at purchase cost, but are of limited impact because part of full capacity capital (K*) (part of the fixed element) is also valued at purchase cost.

Our abstraction from commercial and customer credit on both sides of the balance sheet implies that, throughout the production period (t), corporations, settle any amounts in excess of last years' savings (corporate retained earnings) with additional borrowing. For the sake of simplicity we assume that this happens one off around the end of the production period. However, equation (1.3) below should contain also an element reflecting inventory variation to remain in strict conformity with the balance sheet identity. The underlying assumption for omitting such an element is that over a succession of periods inventory variations add up to roughly zero, or in other words that inventory gravitates around the desired zero level (this result holds in model dynamics). The valuation of fixed capital at purchase cost and inventory at production cost, eliminating thereby non cash flow elements (revaluation reserves) from corporate equity, improves further the performance of equation (1.3) as a measure of corporate debt:, keeping the model simple at the same time.

$L_{t+1} - L_t = (K_t - K_{t-1}) - s_t * NP_t$ (1.3) where: s stands for the rate of corporate retained earnings and NP for net profits.

The following relations hold as well:

$NP_t = s_t * (\Pr_t - i_t * L_t) = s_t * (EQ_t - EQ_{t-1})$ (1.4) where: i the nominal interest rate.

Equation (1.3) is also an «excess demand/ excess supply» expression, when it takes positive values investment exceeds savings and vice versa. Equation (1.4) tells us that net profit equals: gross profit minus interest expense, which together with the rate of savings s(t) determine the evolution in time of corporate equity.

Taking time differences on equation (1.2) given equation (1.3) we arrive to the following expression:

$$\frac{(u_{t+1} - u_t)}{u_t} = (1 - u_t)\frac{(K_t - K_{t-1})}{K_t} \quad (1.5)$$

Equation (1.5) tells us that the rate of growth of capacity utilization depends negatively on the utilization of existing capacity and positively on investment normalized by total capital advanced. Corporations will add capacity when capacity utilization approaches or exceeds unity leading to a decline in the rate of growth of

capacity utilization and at the same time strong growth leads to increased utilization of productive capacity and vice versa. For a relatively stable positive rate of investment capacity utilization will fluctuate around unity and its' rate of growth will gravitate around zero. If profit growth drops to zero and consequently investment is at a standstill then capacity utilization will take a minimum constant value well below unity. A similar equation can be found in (Shaikh 1989).

The last definition is the corporate share of gross profits:

$$y_t = \frac{NP_t}{\mathrm{Pr}_t} = 1 - i_t * \frac{L_t}{\mathrm{Pr}_t} \quad (1.6)$$

Put in words the ratio of net corporate profit to gross profits. This ratio can be expressed also as the difference of the debt service ratio ( i*(L/P) from unity.

Summarizing, the above definitions, we repeat that using a single (basic) rate of profit implies a constant profit margin which is equivalent to abstracting from «counteracting tendencies» in our analysis. Abstracting also from the impact of technology on capacity utilization is equivalent to assuming that its' rate of growth reacts negatively on the utilization of existing capacity and positively on corporate investment. Under this reasoning capacity is fully utilized, on average, in normal accumulation and underutilized in stagnation. We now turn to laying out the main assumptions of the model.

**2. Assumptions:**

We define the growth equation as follows:

$$(K_t - K_{t-1}) \,/\, K_{t-1} = s_t * (r - i_t) \quad (2.1)$$

$$where\ i_t\ the\ curent\ market\ rate\ of\ interest$$

Capital accumulation depends on the rate of profit of enterprise and the rate of retained earnings (s). The rate of interest is treated, in this context, as «opportunity cost» for engaging to or abstaining from active investment. The relation provides insight on how a breakdown in accumulation may incur. If the rate of profit of enterprise shrinks capital accumulation slows down, since industrial capitalists lack the profit incentive to take the risks of production. This may lead to a Marxian «possible crisis» of the first type (breakdown in the reconversion of commodities to money) because capital will remain in monetary form and commodities will pile up. But if capital exiting the production process is meant to repay existing debt or meet previous payments for which it falls short then a crisis of the second type,« the *non- fulfillment of a whole series of payments»* (K. Marx 1861-3, part II ch. 11), may prevail. In the first case money exit circulation and function as a store of value and in the second money from «nominal money of account» turns to a hoard, a «universal commodity» (K. Marx, The Capital VI, ch 3 p.235).

Because, the rate of profit is assumed constant (eq. 1.1) the rate of growth of gross profits equals the rate of growth of capital and equation (2.1) can be expressed as a function of the rate of growth of gross profit as shown below.

$$(Pr_{t+1} - Pr_t) / Pr_t = (K_t - K_{t-1}) \,/\, K_{t-1} \quad (2.1)'$$

We define the rate of interest as follows:

$$i_t = r - a * y_t\ ,\ \ a > 0 \quad (2.2)$$

The rate of interest is a linear function of the rate of profit and the corporate share of gross profits. Equation (2.2) can be easily derived assuming that the share of gross profit is a linear function of the rate of profit of enterprise as shown below:

$$(r - i_t) = a * y_t\ ,\ \ a > 0 \quad (2.2)'$$

Equation (2.2) is also in line with Marx's definition of the determinants of the interest rate *(«it* [the rate of interest-NS] *"depends partly upon the rate of gross profits, partly on the proportion in which these are separated into profits of capital and those of enterprise.* (K. Marx 1894)*)*.

Elaboration of (eq. 2.2) and (eq. 2.2') provides further insight on the interest rate equation. From the definition of (y) (eq.1.6) and equation (2.2) the following result holds$y_{max} = \frac{r}{a}$, since for this value the rate of interest equals zero. It is easy to establish that for positive interest rates$y \leq y_{max}$ must hold. Given that for y=1 debt is zero then for a financial sector to exist $y_{max} \leq 1$ which means that r must be less than a. Substituting the above result in equation (2.2) we find:

$$i_t = r - r * \frac{y_t}{y_{max}} \rightarrow i_t * y_{max} = r * (y_{max} - 1 + i_t * l_t) \rightarrow$$

$$i_t * (y_{max} - r * l_t) = r * (y_{max} - 1)$$

$$\rightarrow i_t = r * \frac{(y_{max} - 1)}{\left(y_{max} - \frac{L_t}{K_{t-1}}\right)} \quad (2.2'')$$

Where $l_t = \frac{L_t}{Pr_t}$ denotes the debt / gross profit ratio[11]. When the rate of interest gravitates around its' maximum, which means,$i_t = r$, the following relation holds:

$$r = r * \frac{(y_{max} - 1)}{\left(y_{max} - \frac{L_t}{K_{t-1}}\right)} \rightarrow \left(y_{max} - \frac{L_t}{K_{t-1}}\right) = (y_{max} - 1) \rightarrow L_t = K_{t-1}$$

Therefore, when current debt is needed to pay for last years' capital, gross profit equals to interest payments. This denotes also, because gross profitability is stagnant (eq. 2.1, 2.1'), that the contribution of corporate equity in production drops to zero in the sense that part of corporate fixed capital remains outside the process of production, seizes to be capital, it is no longer set in motion by living labor. But the accounting measure «shareholders' equity» does not necessarily drop to zero as well. Accounting equity may be reduced because of losses, its' remainder, however, counters land, buildings and machinery which remain unused, but are recorded in the books either at «purchase cost» or at «replacement cost». In other words, corporations, as a reaction to declining profits, downsize their activity to the point where idle fixed assets represent amounts backed by their existing reserves and debt pays for production. In this context financial capital claims for total gross profit since it finances total production. Consequently capacity utilization (eq. 1.2) drops to a minimum as illustrated bellow:

$$u_t = \frac{K_t}{L_{t+1} + EQ_t}$$

$$and\ since\ L_{t+1} = K_t\ and\ profit\ turns\ stagnant$$

$$\rightarrow u_t = \frac{L_{t+1}}{L_{t+1} + EQ_t}\ \ as\ compared\ to\ u_t = \frac{L_{t+1} + EQ_t}{L_{t+1} + EQ_t} = 1$$

In normal accumulation capacity utilization may drop due to fluctuations in demand but debt is mainly reduced instead of equity, because production downsizing releases liquidity in the hands of

[11] For the sake of completion it should be noted also that the modified form of the interest rate (eq. 2.2'') indicates also that for positive interest rates, given a>r which is the plausible choice, the following condition must hold:

$$y_{max} < L_t / K_{t-1}$$

corporations which tempers the debt burden. Although capacity utilization also drops in a crisis debt cannot be retired because all profit is paid as interest therefore corporations cannot build reserves out of savings. Extending this reasoning, if the debt / equity ratio is high, capacity utilization will remain relatively high because there is not enough equity, relative to debt, to back a sharp downsizing in production. This point can prove useful in the discussion of inflation, as well as, the evaluation of the effectiveness of «internal deflation» policies which are used as theoretical justification for fiscal austerity packages implemented by the EU and the IMF. The above are the starting point of a possible extension of this work.

The relations derived so far indicate that when the debt / capital advanced ratio tends to$y_{max}$, y will tend to $y_{max}$ as well. Furthermore, when the debt / capital advanced ratio equals unity the corporate profit share equals zero. This nonlinear negative relation between the share of corporate profits and the debt / capital advanced ratio (eq. 2.2 and 2.2'') can be easily generalized to encompass any values of the two variables. Therefore, our formulation implies that gross profit is distributed between banks and corporations basis the debt required to total capital advanced ratio a measure closely related to the rate of profit[12]. («*This proportion* [of the distribution between profit between interest and profit from enterprise-NS] *again depends upon the competition between the lenders of capital and the borrowers; which competition is influenced, though by no means entirely regulated, by the rate of gross profit expected to be realized.*» (K. Marx 1894, p. 237)).The difference with the previous extract is that, because we keep the rate of profit constant, the expected rate of profit equals the actual.

The illustration has shown that the outline of the rate of interest sited in Capital VIII can be fully described by the definition of the corporate share of gross profit, the linear relation between the later and the rate of profit of enterprise and the limit($y_{max} = \frac{r}{a}$). The question is how this definition reflects financial market relations. In this connection, we now turn to the interpretation of the second parameter of our model the parameter a.

Following Marx we identify credit as the main determinant of the velocity of circulation (K. Marx 1894, p.358) [13]. In this context we attest that the velocity is at a minimum in times of crisis and peaks in times of prosperity. At the same time the profit of enterprise follows the same path. Remembering that we have assumed a linear relation of the profit of enterprise with the corporate share of gross profit, an equation of the following form must hold:

$$\delta * ( v_t - v_{min}) = y_t \; and \; 1/\delta = a \;\; (2.3)$$

Where *v* stands for the velocity of circulation and $v_{min}$ stands for minimum velocity. Therefore the parameter (a) can be viewed as the constant ratio of the difference of the velocity from its' minimum to the share of corporate profits. High values of (a) imply a banking system which will create a big amount of loanable reserves from the deposits in the hands of individual capitalists and corporations, the opposite holds for low values of a.

We can summarize the relations elaborated so far as follows: the debt capital advanced ratio determines the distribution of profit between interest and profit of enterprise, as this ratio tends to unity the rate of interest moves towards the rate of profit and profit of enterprise drops to zero. The reason interest rates explode is that velocity declines due to falloffs in corporate deposits and with it the ability of banks to accumulate money to lend.

Our final assumption (eq. 2.4) determines the rate of savings.

$$s(i_t) = z * i_t = \frac{1}{r} * i_t \; (2.4)$$

[12] This is because any decline in capacity utilization, in our context, is followed by and equal reduction in the return on assets (elaboration of equation 1.2). Therefore, when capacity utilization is at its' minimum return on assets is also at a minimum, which implies that the mass of profit is low. Since the rate of profit is the ultimate regulator of the mass of profit (Grossman.H 1929) then a high debt / capital advanced ratio implies also a low rate of profit.

[13] The chapter begins with a reference to Tooke «The Currency Theory Reviewed» where explicit reference is made on the positive association between velocity and credit.

The usual assumption used in Marxist models is a constant rate of savings, influenced, most probably, by the Keynesian marginal propensity story. We argue that in Marx («schemes of expanded reproduction» (K. Marx 1885)), a variable rate of savings is implied, since it is through variation of savings that an equality of supply and demand can be reached, at least on average over the course of the business cycle. This is also the case in real life, corporations cut back on their distribution policies as a first reaction to declining profitability because of increased interest rates or other sources. Under this line of reasoning we assume that the rate of savings (corporate retained earnings) is a linear function of the rate of interest. Assuming further that for i=r s=1 it follows z=1/r, which reduces the model to two parameters, namely the rate of profit (r) and the structural parameter (a).

We will perform one final elaboration of equation (2.1) in light of equations (2.2) and (2.4), in order to understand the dynamics implied in our assumption on capital accumulation. Substituting (2.2'') and (2.4) into (2.1) the following relation appears:

$$\frac{Pr_{t+1} - Pr_t}{Pr_t} = s_t * r * \frac{y_t}{y_{max}} \quad (2.5)$$

Therefore, the growth equation used in the model is a version of a typical «Marxist equation» where the rate of profit is the dominant factor and growth is internally generated in the sense that savings are reinvested. The last term, which distinguishes our approach from the usual equation, the ratio of the corporate profit share to its' maximum, introduces, together with the definition of savings, interest rate fluctuations influencing the prevailing rate of growth. However, as shown in equation (2.2''), the prevailing debt / capital advanced ratio is the determinant of the distribution of profit between profit of capital and profit of enterprise, high values of this last ratio are closely associated with a low profit rate (see footnote 12). Although, the distribution factor $\frac{y_t}{y_{max}}$, influences growth, it is production which determines the distribution of profit between different classes of capital the latter reacting back on capital accumulation.

Finally, it should be noted that although we will consider only nominal solutions of the model, inflation (to be considered in separate work) can be incorporated in the solutions without altering the conclusions.

### 3. Solution of the Original Model – Stability and Fluctuation Analysis

Through algebraic manipulation (formal derivation is included in appendix 3) the model reduces to the following nonlinear difference map:

$$(\boldsymbol{y_{t+1} - y_t}) = \left((\boldsymbol{2 * r - a}) * \boldsymbol{a} * \boldsymbol{y_t} - \boldsymbol{r^2}\right) * \frac{\boldsymbol{1}}{\boldsymbol{r^2 * (a - r)}} * (\boldsymbol{r - a * y_t})^{\boldsymbol{2}} * \boldsymbol{y_t} \ (\boldsymbol{3.1})$$

This is the basic equation of the model since it determines the time path of y, through which all nominal variables are determined against time. The nonlinear difference equation has the following initial solutions: $y_1 = 0$, $y_2 = \frac{r}{a}$ and $y_3 = \frac{r^2}{(2*r-a)*a}$ (two equal roots). Equation (3.1) also includes an infinite number of secondary positive solutions inside its' secular / chaotic region. We will begin the stability analysis from the initial solutions of the model.

Substituting the initial roots in the derivative (presented in appendix 3) the following stability conditions prevail:

$$\boldsymbol{For\ y_1 = 0}, the\ stability\ condition\ is\ -2 < -\frac{r^2}{a - r} < 0$$

$$which\ holds\ for\ a > r\ \ and\ \ a > r + \frac{1}{2}r^2$$

This solution, which implies$i_t = r$, pictures a depression since for y=0 the rate of growth of profits is also zero (eq. 2.5). The solution is stable when the rate of profit is well below the parameter (a) as indicated by the stability condition. Accumulated retained earnings (savings) cannot take the system out of stagnation, by reducing debt, because all gross profit is used to service the existing debt.

The different types of convergence are summarized below:

For $-1 < 1 - \frac{r^2}{a-r} < 0$ a damping oscillation of y around zero occurs. In the special case $1 - \frac{r^2}{a-r} = -1$ the oscillation has a fixed amplitude with period 1.In all other cases satisfying stability (y) monotonically converges to zero.

$$\boldsymbol{For\ y_2 = \frac{r}{a}} \qquad \frac{dy}{y} = 0$$

$$the\ solution\ is\ semistable\ because\ the\ derivative\ is\ zero$$

This solution, which implies i=0, pictures a state where all corporate profits are consumed and as a result growth is zero because savings are zero. Of course it is an unrealistic state since the interest rate gravitates around zero.

$$\boldsymbol{For\ y_3 = \frac{r^2}{(2 * r - a) * a}}\ the\ stability\ condition\ is:$$

$$-2 < \left(\frac{(r-a) * r}{(2 * r - a)}\right)^2 * \frac{1}{(a-r)} < 0$$

$$which\ holds\ for\ r > a$$

This solution also pictures an unrealistic state of negative leverage with strong growth. Corporations accumulate cash reserves out which they finance their investment plans. This is why for this solution to hold the rate of profit must be greater than the structural parameter (a).

Although the only meaningful initial solution,$y_1$, refers to a stagnant economy there exists a region of parameter combinations which satisfy the following conditions:

$$a < r + \frac{1}{2}r^2 \ \ and\ a > r$$

Inside this region an infinite number of positive solutions for (eq. 3.1) can be identified. Each solution relates to a positive average share of corporate profit and consequently a positive rate of growth. The above imply that there exists a set of parameter combinations which picture a state of the economy where profitability is sufficiently high for the system to grow and growth is either secular or chaotic depending on parameter values and initial conditions. This is the state of normal accumulation where accumulated savings can support capacity utilization adjustments which release capital. This additional liquidity, in the form of corporate deposits, reduces interest rates, by reducing debt and growth resumes. Although, positive excess demand prevails on average over the course of the business cycle influencing the average rate of growth the notion of effective demand is quite different from the keynsian – kaleckian case. More specifically, excess demand (contributing in part to the prevailing average rate of growth) is itself determined by profitability[14]. Furthermore, excess demand is contained by fluctuations in the rate of interest which lead to excess supply on the downside of the economy the opposite motions approximately though not fully canceling each-other. The overall result is a secular stable or semi stable (when chaos prevails) growth path where oscillations reflect variations in demand. We will elaborate on these points in the next section.

[14] This is evident from equation (1.3) and the initial solution$y_1 = 0$, since in stagnation debt remains constant (excess demand equals 0) because both investment and savings are zero.

### 3.1 Equilibrium – Dis- Equilibrium Analysis

As noted in passing in section 1, equation 1.3 can be viewed, in our context, as an excess demand function. This is because we have assumed, in order to keep dynamics simple, that workers do not save and that total capitalist savings are equal to retained earnings. In other words both workers and capitalists consume the total of wages and dividends respectively. Substituting, equations (1.6) and (2.5) we can easily arrive to the following excess demand function:

$$E_t = L_{t+1} - L_t = (K_t - K_{t-1}) - s_t * NP_t$$

$$Where\ E_t\ stands\ for\ excess\ demand.$$

$$Substituting\ (1.6)\ and\ (2.5) on\ the\ right\ hand\ side\ we\ find:$$

$$E_t = \frac{a-r}{r} * s_t * y_t * Pr_t \ (3.2)$$

$$dividing\ both\ sides\ with\ Pr_t\ we\ find:$$

$$e_t = \frac{a-r}{r} * s_t * y_t \ (3.2)'$$

$$Where\ e_t = \frac{E_t}{Pr_t} stands\ for\ the\ rate\ of\ excess\ demand.$$

Equation (3.2') tells us that the rate of excess demand is a linear function of the rate of growth (eq.2.5). This linear relation implies also that the rate of growth of investment equals the rate of growth of savings:

$$I_t = (K_t - K_{t-1}) = a * s_t * y_t * K_{t-1}$$

$$\rightarrow \frac{I_{t+1} - I_t}{I_t} = \frac{s_{t+1} * y_{t+1} * K_t - s_t * y_t * K_{t-1}}{s_t * y_t * K_{t-1}}$$

$$Where\ I_t\ stands\ for\ Investment.\ From\ the\ definition\ of\ savings\ it\ holds:$$

$$S_t = s_t * NP_t = \frac{1}{r} * s_t * y_t * K_{t-1}$$

$$\rightarrow (S_{t+1} - S_t)/S_t = (s_{t+1} * y_{t+1} * K_t - s_t * y_t * K_{t-1})/s_t * y_t * K_{t-1}$$

$$Which\ is\ the\ same\ relation\ as\ above.\ S_t\ standing\ for\ savings$$

This is the dynamic equilibrium condition of (eq. 3.1) which ensures that both the growth trend and the stationary state are stable (semi stable in the chaotic region). Contrary to the traditional approach, this dynamic equality pictures a highly turbulent underlying process where current investment exceeds or underscores current savings, the opposite motions asymptotically tending to cancel each other. This result is reached because we have formulated our model in ratios rather than levels, following the path breaking structure first sited in Goodwin (Goodwin 1967). Equalization of the current levels is almost never reached[15] because unlike the Keynsian – Kaleckian case we don't require equalization of supply and demand by assumption[16]. Furthermore, following Assimakopoullos (Assimakopoulos 1983), we acknowledge that bringing savings to the desired level is a dynamic process during which interest rates will rise and corporations will have to make additional interest payments, therefore any boost in demand carries in it the seed of its' negation. It is only through sufficient profitability that fluctuations in the rate of interest, reflecting fluctuations in demand, will convey

[15] In our formulation current investment equals current savings only in stagnation when both are equal to zero.
[16] A static equilibrium of this form can be reached for a=r which means that the rate of interest is constant. This result can be viewed as a version of the Kaleckian «revolving fund» (Assimakopoulos 1983).

the economy towards a growth path rather than stagnation. In summary it is not the lack of demand which separates growth from stagnation, but the lack of profitability.

### 3.2 Fluctuation Patterns in Normal Accumulation

We will now turn to the analysis of the dynamics inside the normal accumulation region. We can establish that our fourth degree difference equation (eq. 3.1) can be fully approximated by an equation of the form:

$$x_{t+1} = \sigma + \mu * x_t^2 \ (3.3)$$

And for the ruling parameter $\mu$ it holds:

$$\mu = \frac{1}{2} * \frac{r^2}{a - r}$$

This in turn implies that the «Faingenbaum constant[17]» (Feigenbaum 1980) (Brigs 2001), applies for the original equation with control parameter:

$$\pi = 2 * \mu = \frac{r^2}{a - r} \ 3.4$$

Knowing three consecutive limit values of the parameter (π) beyond which the dynamics of equation (3.4) are qualitatively altered and the «Feingenbaum constant»$\delta = 4.6692 = \frac{\pi_n - \pi_{n-1}}{\pi_{n+1} - \pi_n}$, the dynamics of y become fully traceable.

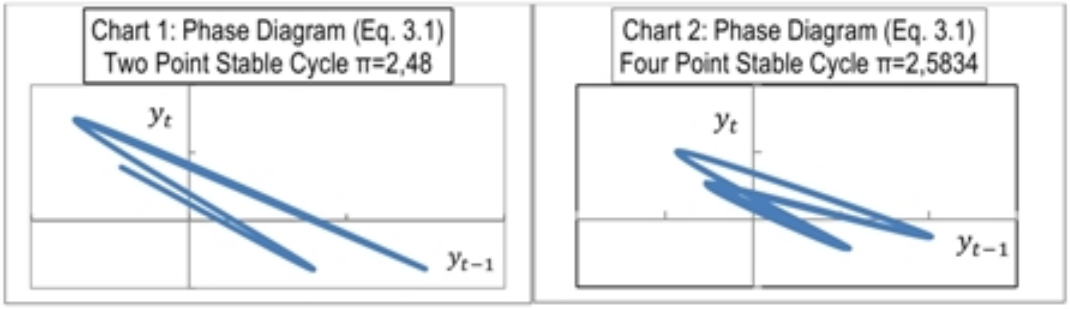
Table 1, bellow, identifies the scales of (π) and the associated dynamics. For (π) greater than two (2) y turns positive and stagnation turns to growth, the motion is a two point stable cycle, if (π) becomes greater than 2.4873 the period doubles and a four point cycle prevails. Whenever the value of (π) exceeds the upper limit, identified in the left hand column of table 1, the cycle period doubles and the next range of values of (π) for which the new dynamics hold becomes shorter. For parameter values greater than 2.6115... the range becomes infinitesimal and any slight change in the parameter value leads to a different set of dynamics, this is the chaotic region of the difference map (eq. 3.1).

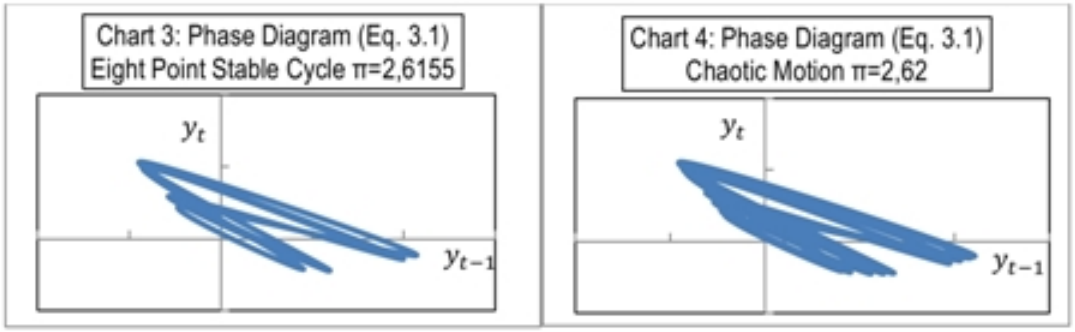
**Table 1: Summary of Stability Condition and Dynamic Motion of (eq.3.1)**

| Scales of Parameter π | Stability and Dynamics |
|---|---|
| 0 < π < 2 | monotonic or oscillatory convergence to zero |
| 2 < π < 2.48573 | two point stable cycles |
| 2,48573 < π <2.58349827 | four point stable cycles |
| 2.58349827 < π <2.611549144 | eight point stable cycles |
| π > 2.611549144 | chaotic motion |

Charts 1-4 below are phase diagrams of $y_t$ against$y_{t-1}$. Each diagram presents the type of motion of y associated to a value of the complex parameter (π) inside the ranges identified on the left hand side column of table 1. The sequence is from top to bottom, omitting the first range, since we will present the stagnation state graphically in the next section.

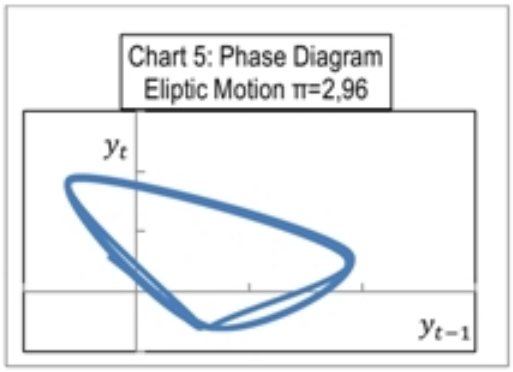
[17] The Feingenbaum constant also referred to as the «silver ratio» is a universal number which holds for all quadratic chaotic difference maps. The number is the constant ratio of the differences of the control parameter values beyond which the cyclical period doubles.




Charts 1 - 4 are derived from simulations executed on equation (3.1) which confirm the algebraic results of Table 1 drawn from the mimic equation (3.3). As the value of parameter (π) (eq.3.4) increases beyond two, which implies that the rate of profit is increased given the value of parameter (a), the system escapes stagnation and enters a region of a two point growth cycle (chart 1). When the rate of profit is increased further ( π> 2.48) the average share of corporate profit is higher, implying stronger growth (eq.2.5) which is associated with more frequent oscillations with smaller amplitude (chart 2). The system undergoes an additional period doubling (chart 3) before entering chaos for (π) values over 2,611 (chart 4). In general, given the value of (a), the higher the rate of profit the stronger the rate of growth which is associated with higher volatility.

Inside the a-periodical (chaotic) region there exist specific values of the control parameter where stable cycles of various periods appear. We present below (chart 5) one such case for the sake of completion.




Besides the analytical findings derived so far, the mathematical exploration of our basic equation (3.1), revealing its' complex dynamics, has further economic inference. Economic data time series, in the model, are secular but not necessarily periodical. This imitates closely the behavior of actual economic data[18]. Therefore, relatively recent literature, arguing that analytical models cannot grasp the complexity of real life and alternatively behavioral patterns should be explored through statistical inference[19] are attempts to side step the unrealistic assumptions of mainstream theory and the formulation of dynamic models in static terms (equalization of levels rather than ratios or trends).

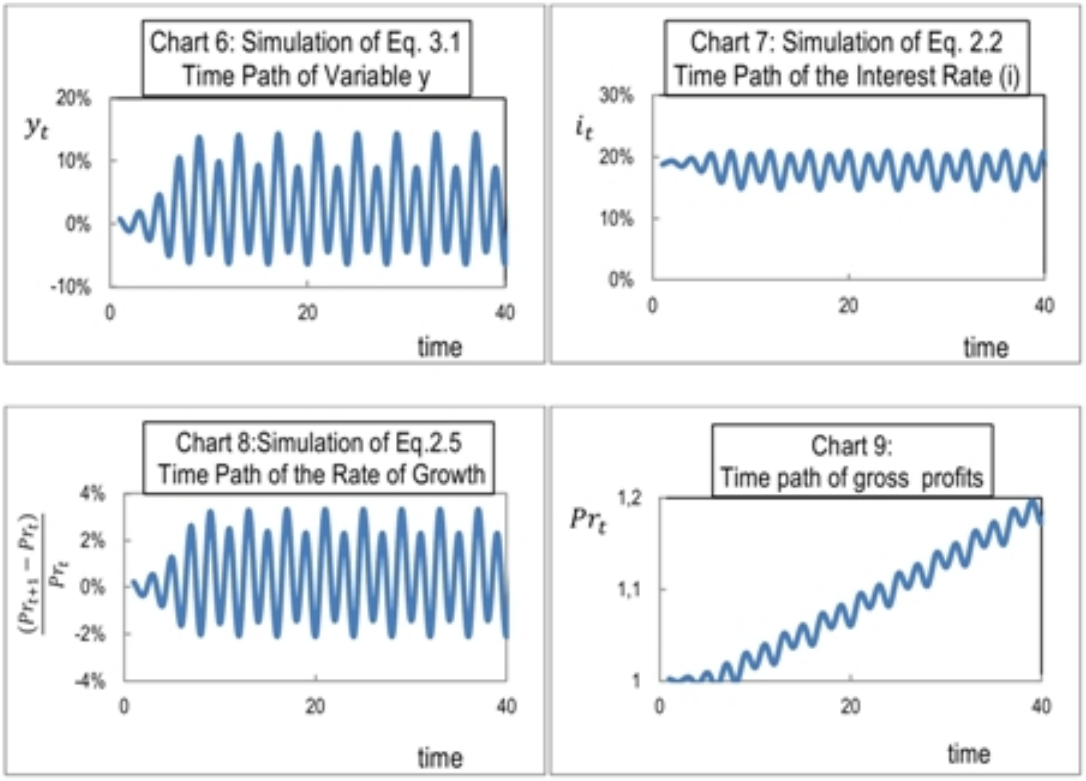
[18] A comprehensive exposition of these findings is part of B. Mandelbrot and R. Hudson (Mandelbrot 2006).
[19]The book by N. Thaleb (N.Taleb 2009) is an extreme and in this sense clear example of this line of reasoning.

Finally, it cannot go without saying, than when it comes down to policy decisions all these reservations are removed and decisions are made on the assumptions criticized for their limitations when explaining the crisis event.

To complete the presentation, of our initial model, we will simulate the time path of the basic variables in normal accumulation and stagnation.

## 4. Simulations

The state of normal accumulation (growth interrupted by recessions) is pictured in charts 6 - 9 below. The parameter values used, to simulate equation (3.1) (chart 6) are a=0.3 and r= 27.1%[20]. The simulated values of y found are substituted in (eq. 2.2) from which the time path of the interest rate is determined (chart 7). By successive substitutions, the time path of rate of growth (eq. 2.5) (chart 8) and the time path of gross profits (chart 9) are determined[21].




The above charts show the various properties of the model. Profits grow persistently (chart 9), interrupted by increased interest rates (chart 7) which accelerate because of increases in debt but fuel also increases in savings through alterations in corporate distribution policy. The overall result is a decline in the rate of growth of profits (chart 8) which reflects a stronger decline in the corporate share of gross profit (chart 6). As a reaction, corporations downsize production supported by accumulated savings (retained earnings) (eq. 1.5 elaborated in section 2). Because the rate of profit is sufficiently high (the organic composition of capital is relatively low) the funds released are not used up as means of payment but reduce debt bringing down the rate of interest and growth is restored. Fluctuations in this state can be viewed as variations in aggregate demand, since equation (1.3), elaborated in section 3.1, can be read as the familiar relation investment minus savings. But these fluctuations convey towards a growth path because the rate of profit is sufficiently high allowing efficient adjustments in debt.

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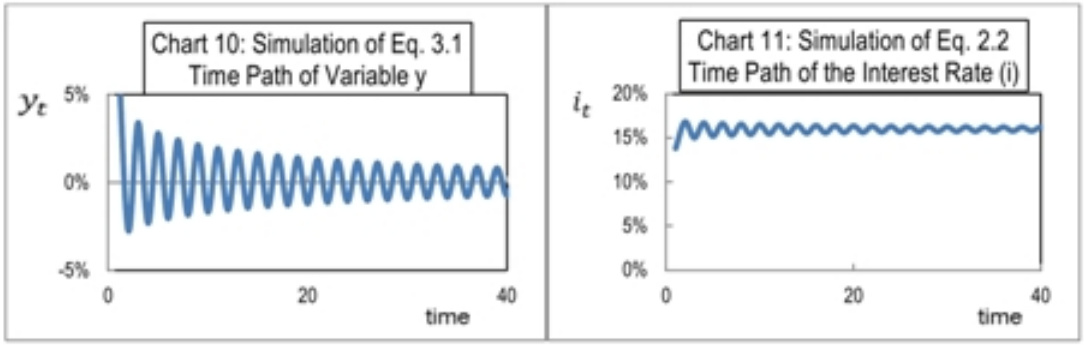
[20] To connect with the findings in section 3.2 the value of the control parameter is: $\pi = \frac{r^2}{a-r} = 2.532$ and the anticipated dynamics a four point cycle confirming the findings in table1.

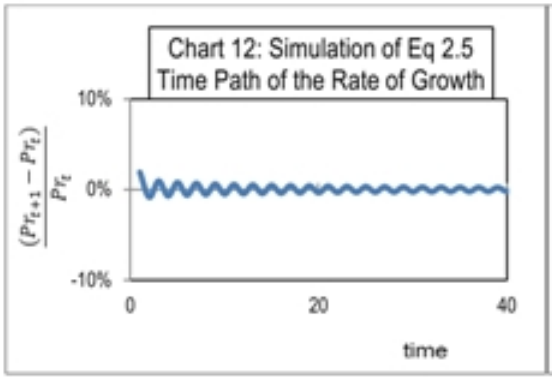
[21] In this last simulation (chart 9) an initial value (arbitrarily chosen to equal unity) for gross profit was included in order to determine the time path.

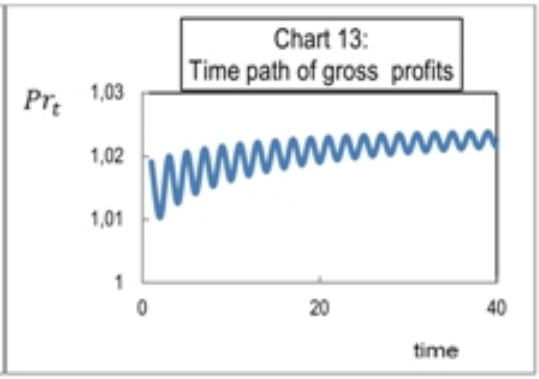
Stagnation prevails when the rate of profit is below the limit indicated by the stability condition of solution $y_1 = 0$ (section 3). Charts 10-13 below are simulations of the same equations as charts 6-9 above the only difference is that the rate of profit is now r = 15.5 % instead of 27% previously used.










The share of corporate profits (chart 10) oscillates around zero because the rate of interest gravitates around the rate of profit (chart 11), which implies that net profits are zero (y=0). As a result the rate of growth is also zero (chart 12) and profitability turns stagnant (chart 13). Sufficient debt reductions are unfeasible since corporations consume all gross profit to service existing debt and are unable to accumulate reserves. Any variations in the rate of growth resulting from interest adjustments, reflecting alterations in capacity utilization, are shortly reversed because profit rates are too low.

Besides the dominating influence of the rate of profit on growth, the above illustration is indicative of the limited effect of bank refinance/recapitalization on output and employment. In stagnation debt is meant to pay for production costs (section 2) not to support investment, banks which monitor the performance of their clients are aware of this. Therefore banks will ask for first class collateral just to revolve the existing credit and with same rational for additional collateral to extend the credit lines. Even if complete recapitalization is implemented banks will end up «sitting» on the liquidity because, due to production downsizing, corporate activity, as we have shown (section 2), is barely sufficient to cover the existing debt. It is only after relatively stable increases in corporate deposits that credit will expand again, but this will require a significant impairment of capital which banks have no incentive to initiate.[22]

**5. Bank deregulation and the present depression:**

We claim that the conclusions reached so far have general validity, moreover they directly apply to the policies followed to exit the previous great depression of the 70s also referred to as the

[22] For instance banks could offer to turn part of the debt to equity but they will do this only when the existing debt is not properly secured invoking concerns about the security of the whole debt commitment. Capital impairment, which in this case will happen through the dilution of the old shareholders, will take place as each problematic corporate case becomes evident. However, in our model corporations will hold on average adequate collateral to cover debt at the beak even level, therefore this process is not expected to be undertaken in great extent.

«great stagflation». The deregulation of the labor market and the demolition of the welfare state stabilized profit rates which were falling in the preceding postwar decades. But profit rates remained inside the «stagnation region» imitated above in solution$y_1$ section 3. Because r could not increase, justified by «market self-regulation», (a) was increased. In other words, banks were allowed to extend credit to unprecedented levels based on moderate amounts of corporate deposits.

Of course, this is a simplified version of bank deregulation in order to fit our context of a banking system offering a single type of loan (corporate loans) and a single liability (corporate deposits). In reality a huge variety of financial assets were issued extending finance to corporations, households and sovereigns. All cases, however, shared a common treatment, the creation of secondary, regulated or over the counter, markets where these assets were actively traded, the trading stimulated by derivative, mainly bilateral (forward), contracts. This way banks were able to fuse their risk on the whole society (corporations, workers, capitalists, the state etc.) and to extend their balance sheets further. The goal was simple, irrespective of the complexity of assets and financial intermediaries, the creation of increasing liquidity which together with low central bank intervention rates kept market interest rates low permitting further extension of credit because of low debt service costs.

We argue that the dynamics of this practice can be traced in our simple context because the objective is the same irrespective of the classes of assets and debt recipients. This policy was initiated to create a positive profit of enterprise and resume growth because the rate of profit was low. However, corporate behavior is also modified in this environment. Low interest rates producing a positive profit of enterprise pushes corporations to extend their balance sheets with credit in order to maximize returns on their own capital. The financial sector, on the other hand, boosts the velocity of circulation, because of looser regulations and growth resumed irrespective of the prevailing rate of profit. But, as we will show, the joy cannot last forever since the prevailing rate of profit bounds the extension of financial capital.

This aspect of financialization is picked in (Lapavitsas 2009) but leads him to the wrong conclusion: that financialization is not necessarily the result of weak production it can result at any profit rate because of the increased autonomy of the financial sector. This is half the truth, for the financial sector to grow a corporate sector eager to take additional debt is required[23]. And as we have argued the corporate sector has higher debt needs the lower the rate of profit. Corporations took up additional debt since by inflating debt their otherwise low equity returns were increased because of low debt service costs.

These points will become evident by reconstructing our model assuming constant interest rates. Of course interest rates were never constant between 1980–2007, however they kept declining with very small volatility for most of the period and were treated as a nonissue in corporate investment planning, especially in the decade following the millennium.

Our accumulation function (eq. 2.5) remains valid in the constant interest rate environment, although the linear relation between the share of corporate profit and profit of enterprise (eq. 2.2) is broken. When interest rates are relatively stable equation 2.2 is replaced by a linear relation of the share of corporate profits and return on equity. We prove this here below:

$$We\ repeat\ our\ growth\ equation\ (2.5):$$

$$\frac{K_t - K_{t-1}}{K_{t-1}} = s_t * r * \frac{y_t}{y_{max}}$$

$$Since\ the\ rate\ of\ interest\ is\ assumed\ constant\ the\ rate\ of\ savings\ \ (eq\ 2.4)$$

$$is\ also\ constant\ and\ (2.5)\ is\ modified\ as\ follows:$$

$$\frac{K_t - K_{t-1}}{K_{t-1}} = s * a * y_t \rightarrow K_t - K_{t-1} = i^* * a * (EQ_t - EQ_{t-1})$$

$$Where\ i^*\ is\ the\ constant\ interest\ rate\ assumed$$

$Reformulating\ the\ above\ relation\ in\ continuous\ time\ in\ order\ to\ integrate\ the\ relation$[24].

[23] The same argument can be extended to encompass households, sovereigns etc.

[24] This means that we assume an infinitesimally small production period.

We can rewrite as follows:

$$\frac{dK}{dt} = i^* * a * \frac{dEQ}{dt}$$

$$integrating\ on\ both\ sides\ and\ ommiting\ the\ coefficient\ of$$

$$the\ following\ relation\ holds:$$

$$K = i^* * a * EQ$$

$$Dividing\ both\ sides\ with\ NP\ and\ rearanging\ we\ find:$$

$$ROE = r * i^* * a * y\ (4.1)$$

Substituting (eq. 4.1) in (2.5) we find that our growth equation in a constant interest rate environment reads as follows:

$$\frac{K_t - K_{t-1}}{K_{t-1}} = \frac{1}{r^2} * ROE_t\ (4.2)$$

From equations (4.1), (eq. 4.2) and constant interest, the limit of the growth trend is expressly determined from the definition of (y) (eq. 1.6):

$$1 - i^* * \frac{L_t}{Pr_t} = 0 \rightarrow \frac{L_t}{Pr_t} = \frac{1}{i^*}\ \ (4.3)$$

When the debt gross profit ratio equals to the reciprocal of the rate of interest all gross profit will be paid out as interest (y=0) and growth stops. In other words profitability poses a barrier on growth by limiting the maximum debt burden, given the rate of interest. Furthermore, given the maximum debt burden, the lower the mass of profit (which implies a lower rate of profit) the lower the interest required for growth to prevail (positive ROE). At the same time (a) must increase so that the velocity of circulation (eq. 2.3) will rise, making additional debt available to corporations without altering the rate of interest. This point will be clarified from the stability condition of the modified time difference map of (y) which follows.

Following algebraic manipulation, presented in Appendix 4, the following difference map of y is derived in constant interest rates:

$$y_{t+1} = \left(1 - \frac{ari^*}{\left(r^2 + ari^* - (a-r)i^{*2}\right)} y_t\right) \frac{\left(r^2 + ari^* - (a-r)i^{*2}\right)}{r^2} y_t (4.4)$$

Equation (4.4) can be reduced further to read as follows:

$$\boldsymbol{z_{t+1} = (1 - z_t) * \varphi * z_t\ \ (4.4')}$$

$$Where: z_t = \frac{a * i * r}{a * r * i - (a-r) * i^2 + r^2} * y_t\ and\ \varphi = \frac{(a * r * i - (a-r) * i^2 + r^2)}{r^2}$$

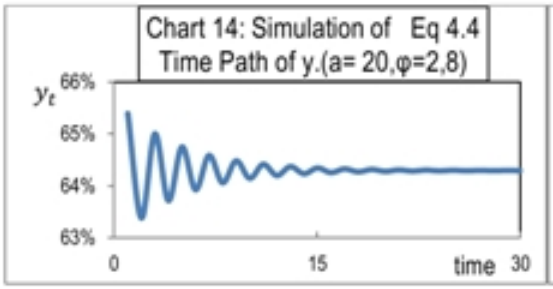
The nonlinear difference equation (4.4') is known as the «logistic map», initially introduced by the 19th century Belgian biologist Pierre Francois Venhulst in differential form and is used to describe population dynamics. In 1975 the biologist Robert May (May 1975) presented the equation as a difference map and explored its' complex chaotic dynamics. The values of the control parameter $\varphi$ and the relevant dynamics are summarized in table 2 which follows:

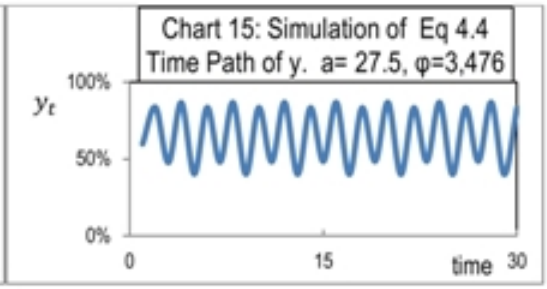
| Table 2: Parameter Scales and Dynamic Motion of EQ. 4.4' | |
|---|---|
| **Parameter Scale** | **Dynamic Motion** |
| 1<φ<2 | monotonic convergence |
| 2<φ<3 | oscillatory convergence |
| 3<φ<3,4494 | two point cycle |
| 3,4494<φ<3,54 | four point cycle |
| 3,54<φ<3,57 | 8,16,32 period cycles |
| 3,57<φ<3,8284 | Chaos |
| 3,8284<φ<4 | eliptic motion |
| φ>4 | system collapse |

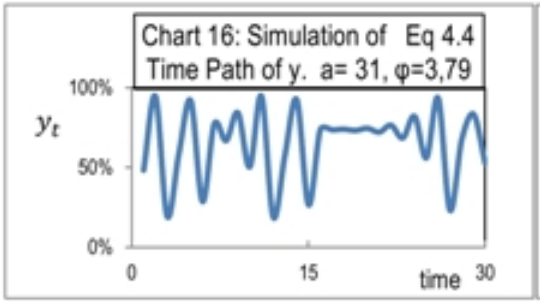
Table 2 is constructed with the same rational as Table 1 in section 3.2. The difference is that because the logistic map has been extensively analyzed the scales have been taken from the relevant literature instead of derived. We have omitted also the values of the control parameter (φ) between zero and unity because they do not apply to plausible values of the basic parameters: (r), (a), (i)[25]. For parameter values $1 < \varphi < 3$ the model will converge (either monotonically or through dumping oscillations) towards a fixed point$(1 - \frac{1}{\varphi})$. For $3 < \varphi < 4$ the model undergoes various period doubling sub-segments, chaotic and elliptic motion regions. Finally, for parameter values greater than four (4) the system collapses.

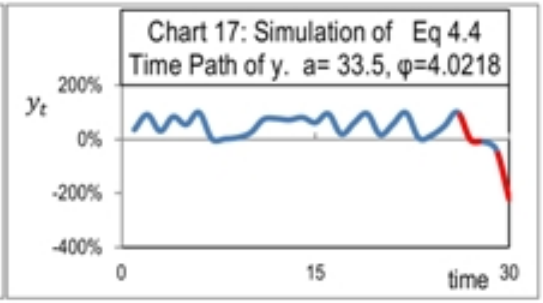
Given a positive constant rate of interest and the rate of profit the control parameter (φ) is a positive function of the structural parameter (a) (eq. 4.4'), but (a) cannot increase indefinitely it is bounded by the rate of profit for positive interest rates. Nonetheless, the model implies positive rates of growth for parameter values less than four (4). The reasonable question which arises is why corporations and banks will extend credit further although they enjoy growing profitability? The reason is simple, growth is stronger the greater the value of (a) (Eq. 4.2, 4.4). Both corporations and banks shared strong incentive to extend credit because both interest and profit rates were low. This was the case for a long period of time on a world scale, regulations were relaxed and interest rates fell bringing the system closer and closer to collapse until it prevailed in 2007.

Charts 14-17 are simulations of the time path of (y) (eq. 4.4) for different values of (φ) resulting from variations in (a) with constant profit and interest rates (i=1%, r=10%):




[25] For parameter values $0 < \varphi < 1$ the model converges to zero i.e. to a stagnation state like the one presented in section 4.

Chart 14 pictures a damping oscillation although interest rates are low bank deregulation has not advanced enough for credit to extend further. In the next graph (chart 15) credit is further extended, this implies stronger average growth (eq. 4.2) because (a) has sufficiently increased although average (y) is slightly lower. Further increase in (a) puts (eq. 4.4) in the chaotic region (chart 16), strong a-periodical oscillations prevail, however the variable keeps returning asymptotically to the average value. Finally, chart 17 pictures the model breakdown ($\varphi>4$), although the system experiences chaotic oscillations for some time, suddenly (y) collapses and growth (eq. 4.2) turns to a free-fall (time path segment painted red). This happens because credit is slightly overextended and corporations experience slight losses which however persist. because the rate of profit is low. Suddenly production collapses, corporations experience severe losses and banks soon find themselves with a deteriorating asset side and sharply declining deposits, the velocity of circulation collapses because everyone is trying to secure his money and whole economy is trapped in a «death spiral».

To stop the spiral, in these circumstances, is the clear part, intervention rates are reduced to zero and bank finance is provided by the central bank to avoid failures. However, market interest rates will not follow intervention rates in this environment because the rate of profit remains low meaning that corporate deposits will not be restored. Banks on the other hand will keep asking for funds until their asset side seizes to deteriorate. The economy will end up in a stagnation state similar to the one presented in section 4.

**Conclusion:**

The models laid out above are meant to support two basic theoretical points:

1) In Marx, crisis prevails when the rate of profit is so low that corporate reserves are not sufficient to restore the liquidity of the banking system. Debt outstanding becomes too high relative to the surplus value to be appropriated by the corporate sector, interest rates explode and growth turns to stagnation. This result is derived from Marx's theory of corporate investment, interest and money which are integrated in the concept of profit of enterprise. In explaining the present crisis, this result is of importance because it takes the explanatory focus from the celebrated dynamics of the rate of profit, which do not apply to the events preceding the current depression, to the economic factors which determine the passage from normal accumulation to depression, where the rate of profit prevailing is the dominant element. This last issue keeps the core of Marx's argument intact because although profitability remains the driving force of accumulation the anticipated dynamics of the rate of profit are not a prerequisite for growth to turn to stagnation.

2) The second point has to do with an attempt to provide some analytical insight on aspects of the phenomenon of financialization of capital. We have shown, in the context of our original model, that financialization in the sense of increasing dependence of capital accumulation on the money creation powers of the banking sector is inversely proportionate to the rate of profit. The lower the rate of profit the higher the leverage needed. Furthermore, the share of surplus value appropriated by the financial sector is higher the lower the rate of profit. But, this is half the truth, the financialization of all aspects of economic life, by establishing secondary and derivative markets, has no historical precedent it is a new aspect of contemporary capitalism. We argue that, financialization was the result of a strategy which emerged as a response to persistent low profit rates, the objective being to create a positive profit of enterprise by suppressing interest rates. This policy gave increasing autonomy to the financial sector and modified the behaviour of both corporations and banks. Extending balance sheets though leverage became the primary strategy, this on one hand promoted growth, but on the other kept increasing the fragility of the system until its' collapse in 2007. We showed that low profitability is a prerequisite for financialization, but at the same time the prevailing rate of profit poses a limit to financial expansion.

These theoretical results have also important policy implications, arising from the mere fact that under our reasoning and irrespective of the special actualities which led to the current crisis, it classifies under the category of great depressions. The system has to undergo severe restructuring to restore growth.

The collapse in 2007, followed by high interest rates and tight credit, triggered sharp reductions in output and employment. Securitization of financial capital in these circumstances simply led to extensive hoarding (this is the case in the U.S.). This would be case even if the amounts advanced were in excess of the funds needed to support the existing exposure of the banking sector. As we have shown the available collateral in the hands of corporations is barely sufficient to cover their outstanding debt because production is downsized in depressions. Banks have no incentive to bring capacity utilization to the normal level through unsecured credit, risking their own capital in this regard. It is only after deposits will commence picking up again that credit will begin to expand. But this requires the restoration of profitability which in turn requires the impairment of weak capital and the operation of counteracting tendencies. Of the later the suppression of wages in the name of balanced budgets and fiscal austerity, appears the one most broadly implemented, especially in Europe. This means that even when the system recovers the needs of the many will be so hardly suppressed that it will take decades before reaching pre-crisis levels.

Policy alternatives supporting political and social activism are desperately needed in the present economic political and social environment and the understanding of the causes of the depression is a prerequisite for their formulation. Hopefully, this paper made a small contribution in this direction.

Extensions of our formulation are possible incorporating an alternative approach of inflation, sovereign deficit and debt, as well as alternative fiscal policy and policy evaluations of fiscal austerity programs imposed by the EU and the IMF on Southern Europe. This will be the focus of future work.

**Appendix 1:**

We will prove below that if the profit margin is constant then, given the labor market conditions the prevailing rate of profit will equal the basic rate. The additional notation used is: W = wages, M= materials, Depr = depreciation, INV= inventory, m= profit margin on costs, CC= constant capital.

$$The\ profit\ margin\ is\ defined\ as\ follows:$$

$$\frac{Pr_{t+1}}{W_t + M_t + Depr_t} = m_{t+1}$$

$$it\ also\ holds:$$

$$Sales_{t+1} = (W_t + M_t + Depr_t - INV_{t+1}) + Pr_{t+1}{}^{26}$$

$$\rightarrow \frac{Sales_{t+1}}{Pr_{t+1}} = \frac{1}{m_{t+1}} - \frac{INV_{t+1}}{Pr_{t+1}} + 1 \rightarrow \frac{Sales_{t+1}}{Pr_{t+1}} = \frac{1 + m_{t+1}}{m_{t+1}} - \frac{INV_{t+1}}{Pr_{t+1}}$$

$$Furthermore\ assuming\ a\ constant\ rate\ of\ depreciation\ (\gamma)\ and\ remembering\ that\ the\ organic\ composition\ of\ capital\ (d) is\ constant\ it\ follows:$$

$$Depr_t = \gamma * CC_t\ and \frac{W_t}{M_t + CC_t} = d \rightarrow \frac{W_t}{M_t + \frac{1}{\gamma} * Depr_t} = d$$

$$and\ K_t = (1 + d) * \left(M_t + \frac{1}{\gamma} * depr_t\right) it\ follows:$$

$$Sales_{t+1} = dM_t + \frac{d}{\gamma} Depr_t + M_t + Depr_t + \frac{1}{\gamma} Depr_t - \frac{1}{\gamma} Depr_t - INV_{t+1} + Pr_{t+1}$$

$$= K_t - \frac{1 - \gamma}{\gamma} * Depr_t - INV_{t+1} + Pr_{t+1}$$

$$\rightarrow \frac{S_{t+1}}{Pr_{t+1}} = \frac{K_t}{Pr_{t+1}} - \frac{1 - \gamma}{\gamma} * \frac{Depr_t}{Pr_{t+1}} - \frac{INV_{t+1}}{Pr_{t+1}} + 1$$

$$\rightarrow \frac{S_{t+1}}{Pr_{t+1}} - 1 + \frac{INV_{t+1}}{Pr_{t+1}} = \frac{K_t}{Pr_{t+1}} - \frac{1 - \gamma}{\gamma} * \frac{Depr_t}{Pr_{t+1}} \rightarrow$$

$$\rightarrow \frac{1}{m_{t+1}} = \frac{K_t}{Pr_{t+1}} - \frac{1 - \gamma}{\gamma} * \frac{Depr_t}{Pr_{t+1}}$$

$$\rightarrow \frac{\gamma * Pr_{t+1} + (1 - \gamma) * Depr_t}{\gamma * m_{t+1} * Pr_{t+1}} = \frac{1}{r_{t+1}} \rightarrow \boldsymbol{\gamma * \frac{Pr_{t+1}}{K_t} + (1 - \gamma) * \frac{Depr_t}{K_t} = \gamma * m_{t+1}}$$

$$And\ since\ the\ ratio\ of\ Depreciation\ to\ Kapital\ Advanced\ is\ expected\ constant$$

[26] We use the business accounting definition, the term in brackets is the accounting term "cost of goods sold". Sales are the sum of cost of goods sold and gross profit

$because\ the\ organic\ copmposition\ of\ capital\ is\ constant\ it\ follows\ that\ if\ the\ profit\ margin\ is\ constant\ then\ the\ prevailing\ profit\ rate\ is\ also\ constant\ and\ equal\ to\ the\ basic\ rate.$

**Appendix 2:**

We derive the tine difference equation of capacity utilization (equation 1.5 in the text).

$$\frac{K_t}{L_{t+1} + EQ_t} = u_t$$

$$\rightarrow K_t - K_{t-1} = (u_{t+1} - u_t) * (L_{t+1} + EQ_t) + u_t * (L_{t+1} - L_t + s_t * NP_t)$$

$$\rightarrow K_t - K_{t-1} = \frac{(u_{t+1} - u_t)}{u_t} * K_t + u_t * (K_t - K_{t-1})$$

$$\rightarrow (1 - u_t) * \frac{K_t - K_{t-1}}{K_t} = \frac{(u_{t+1} - u_t)}{u_t}$$

$$\rightarrow \frac{K_t - K_{t-1}}{K_t} = \frac{(u_{t+1} - u_t)}{u_t * (1 - u_t)} \rightarrow . \frac{(u_{t+1} - u_t)}{u_t} = (1 - u_t) * \frac{K_t - K_{t-1}}{K_t}$$

**Appendix 3:**

Derivation of basic difference map (eq. 3.1). Letting $\frac{L_t}{Pr_t} = l_t$ and since from the definition of y (equation 1.6 in the text) it holds $l_t = \frac{1-y_t}{i_t}$, taking time differences we find:

$$l_{t+1} - l_t = -(y_{t+1} - y_t)\frac{1}{i_t} + \frac{a}{i_t}(y_{r+1} - y_t)l_t = (al_t - 1)(y_{t+1} - y_t) * \frac{1}{i_t}$$

Furthermore, taking time difference on the definition of $l_t$ the following relation holds as well:

$$l_{t+1} - \ l_t = \frac{L_{t+1} - L_t}{Pr_t} - \frac{Pr_{t+1} - Pr_t}{Pr_t} l_t = \frac{(K_t - K_{t-1}\ )}{r * K_{t-1}} - s_t \frac{NP_t}{Pr_t} - \frac{Pr_{t+1} - Pr_t}{Pr_t} l_t$$

$substituting\ the\ growth\ and\ debt\ time\ difference\ equations\ (eq\ 2.1, 1,3)\ in\ the\ abov$

$$l_{t+1} - \ l_t = \frac{1}{r^2} i_t(ay_t) - \frac{1}{r} i_t y_t - \frac{1}{r} i_t(ay_t) l_t = \left(\left(\frac{1}{r} - l_t\right)(ay_t) - y_t\right)\frac{1}{r} i_t$$

Equalizing the two forms, substituting $l_t$ with $\frac{1-y_t}{i_t}$ and solving for the time difference of $y_t$ we find:

$$(a * l_t - 1) * (y_{t+1} - y_t) * \frac{1}{i_t} = \left(\left(\frac{1}{r} - l_t\right) * (a * y_t) - y_t\right) * \frac{1}{r} * i_t$$

$$(y_{t+1} - y_t) = \left(\left(\frac{1}{r} - l_t\right) * (a * y_t) - y_t\right) * \frac{1}{r * (a * l_t - 1)} * i_t^2$$

$$since\ (a * l_t - 1) = a * \frac{1 - y_t}{i_t} - 1 = \frac{a - a * y_t - r + a * y_t}{i_t} = \frac{a - r}{i_t}$$

$$and\ also\ \frac{1}{r} - l_t = \frac{1}{r} - \frac{1 - y_t}{i_t} = \frac{r - a * y_t - r + r * ay_t}{r * i_t} = \frac{(r-1) * y_t}{r * i_t}$$

$$(y_{t+1} - y_t) = ((r-1) * (a * y_t) - r * i_t) * \frac{1}{(a-r) * r^2} * i_t^2 * y_t$$

Substituting equation 2.2 for $i_t$ we get:

$$\boldsymbol{(y_{t+1} - y_t) = \left((2 * r - a) * a * y_t - r^2\right) * \frac{1}{r^2 * (a-r)} * (r - a * y_t)^2 * y_t\ (3.1)}$$

The derivative is:

$$\frac{dy_{t_1}}{y_t} = \left((2r-a)ay - r^2\right)\frac{1}{r^2(a-r)}(r-ay_t)^2 + \left((2r-a)a\right)\frac{1}{r^2(a-r)}(r-ay_t)^2 y_t \quad - a\left((2r-a)ay - r^2\right)\frac{1}{r^2(a-r)}2(r-ay_t)y_t$$

**Appendix 4:**

We will derive the time difference equation of y (eq. 4.4, 4.4' in the text) in constant interest rate as follows:

$$l_{t+1} - l_t = -(y_{t+1} - y_t) * \frac{1}{i}$$

$$l_{t+1} - \ l_t = \left(\left(\frac{1}{r} - l_t\right) * a - 1\right) * \frac{1}{r} * i * y_t = \left(\left(\frac{1}{r} - \frac{1 - y_t}{i^*}\right) * a - 1\right) * \frac{1}{r} * i * y_t$$

Equalizing the two forms we get:

$$-(y_{t+1} - y_t) * \frac{1}{i} = \left(\left(\frac{1}{r} - \frac{1 - y_t}{i^*}\right) * a - 1\right) * \frac{1}{r} * i * y_t$$

$$\rightarrow -(y_{t+1} - y_t) = \left(\left(i - r(1 - y_t)\right) * a - r * i\right) * \frac{i}{r^2} * y_t$$

$$\rightarrow -(y_{t+1} - y_t) = \left((a-r) * \frac{i^2}{r^2} - \frac{a}{r} * i + \frac{a}{r} * i * y_t\right) * y_t$$

$$\rightarrow y_{t+1} = \left(1 - \frac{a * r * i}{(r^2 + a * r * i - (a-r) * i^2)} * y_t\right) * \frac{(r^2 + a * r * i - (a-r) * i^2)}{r^2} y_t$$

Letting:

$$z_t = \frac{a * i * r}{a * r * i - (a-r) * i^2 + r^2} * y_t \qquad \varphi = \frac{(a * r * i - (a-r) * i^2 + r^2)}{r^2}$$

And substituting we find:

$$z_{t+1} = (1 - z_t) * \varphi * z_t$$

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